Scattering Models for Reverberation Problems

Issues to be addressed

Relatively standard roughness spectra will be used for the surface and bottom. These are 2-D spectra appropriate for 3-D reverberation problems. For the 2-D reverberation problems corresponding 1-D spectra are needed, and models for those will be specified.

For both the surface and bottom and for both 2-D and 1-D surfaces, bistatic scattering models are needed, and these include monostatic backscattering as a special case. Most likely these will be used only for bistatic backscattering, and even when 2-D roughness is being treated, only in-plane bistatic backscattering will likely be utilized in reverberation modeling. However, models for more general bistatic scattering will be given.

In addition to bistatic scattering models, coherent reflection loss models for both the surface and bottom are needed for both 2-D and 1-D surfaces.

For bistatic backscattering, low grazing angle scattering will be dominant for the reverberation except at very short times. Because of this, the use of perturbation theory for modeling the bistatic backscattering processes will be assumed. At our highest frequency of 3.5 kHz, perturbation would not be accurate over the entire range of incident and scattering angles, especially for forward scattering at higher grazing angles. But, at the low grazing angles of primary interest, perturbation theory should be adequate for bistatic backscatter modeling up to 3.5 kHz for sensible ranges. Perturbation theory is also very convenient in that relatively simple analytic expressions can be given for the needed models. It may turn out that at very short times where higher grazing angles come into play, the limitations of perturbation will become significant. This is one area where comparisons with full numerical solutions (not employing perturbation theory) would be especially useful. And, this is the regime where numerical solutions are most readily carried out.

It is less clear that perturbation theory would be adequate for coherent reflection loss models up to 3.5 kHz. This is so because perturbation typically fails first near the forward direction as the roughness or frequency is increased, and the reflection loss is obtained by integrating the scattered energy over all outgoing directions, which is dominated by the forward region. Because of this, coherent reflection loss models will be based on the more accurate small slope approximation (SSA). The SSA is more complicated to employ, but is worth the trouble for the coherent reflection loss. Fortunately, it is believed that SSA models for bistatic backscattering can be avoided for the problems of interest.

Surface bistatic scattering for 2-D surfaces

For a 2-D rough surface (for the 3-D reverberation problem), we write the bistatic scattering strength as

$$SS = 10\log_{10}\sigma_{\mathrm{2D}} \;, \qquad (1)$$

where σ_{2D} is the bistatic scattering cross section per unit area per unit solid angle and is dimensionless.

The (lowest-order) perturbation theory expression for the bistatic scattering cross section is given by

$$\sigma_{\mathrm{2D}} = 4k_{iz}^2 k_{sz}^2 P_{\mathrm{2D}}(\boldsymbol{K_i} - \boldsymbol{K_s}) = 4k^4 \sin^2\theta_i \sin^2\theta_s P_{\mathrm{2D}}(\boldsymbol{K_i} - \boldsymbol{K_s}) . \qquad (2)$$

In this expression, k is the acoustic wavenumber in the water, and we write the acoustic wave vector as

$$\boldsymbol{k} = \boldsymbol{K} + k_z\hat{\boldsymbol{z}} \;, \qquad (3)$$

where $\boldsymbol{K}$ denotes the horizontal component of the wave vector. The angles in (2) are the incident and scattered grazing angles. Finally, $P_{\mathrm{2D}}(\boldsymbol{K})$ is the 2-D surface roughness spectrum, normalized such that the mean square surface height h^2 is given by

$$h^2 = \int P_{\mathrm{2D}}(\boldsymbol{K}) d^2K . \qquad (4)$$

We take $P_{\mathrm{2D}}(\boldsymbol{K})$ to be given by an isotropic Pierson-Moskowitz spectrum for a fully developed sea. Thus,

$$P_{\mathrm{2D}}(\boldsymbol{K}) = \frac{\alpha}{4\pi K^4}\exp(-K_L^2/K^2), \quad (5)$$

where $K = |\boldsymbol{K}|$, $K_L = \sqrt{\beta}\, g/U^2$, $g = 9.81$ m/s^2, $\alpha = 0.0081$, and $\beta = 0.74$.

In the original Pierson-Moskowitz formulation, U is the wind speed at a height of 19.5 m, not the present standard of 10 m. This refinement will be ignored, and the wind speed of 10 m/s will be used directly in these expressions.

Using (4) and (5) one finds that the rms surface height h is given by

$$h = \frac{1}{2}\sqrt{\frac{\alpha}{\beta}}\,\frac{U^2}{g} \; . \quad (6)$$

For $U = 10$ m/s, this gives $h = 0.53$ m, and then the significant wave height H, approximately given by $H \cong 4h$, is 2.13 m.

When the argument of $P_{\mathrm{2D}}(\boldsymbol{K})$ is $\boldsymbol{K_i} - \boldsymbol{K_s}$ as in (2), then K^2 in (5) is replaced by

$$k^2[(\cos\theta_i - \cos\theta_s\cos\phi)^2 + \cos^2\theta_s \sin^2\phi]$$

where ϕ is the asimuthal angle between $\boldsymbol{K_i}$ and $\boldsymbol{K_s}$. Thus, for specular scattering, $\phi = 0$, and for in-plane bistatic backscattering, $\phi = \pi$. Therefore, for in-plane bistatic backscattering, K^2 in (5) is replaced by $k^2(\cos\theta_i + \cos\theta_s)^2$ and for monostatic backscattering by $4k^2\cos^2\theta_i$.

Surface bistatic scattering for 1-D surfaces

The first step is to obtain a 1-D roughness spectrum. There is not a unique prescription for obtaining a 1-D spectrum from a 2-D spectrum in order to yield reverberation that most closely matches that found with the initial 2-D spectrum. The choice taken here is to define the 1-D spectrum (for positive and negative K_x) as

$$P_{\mathrm{1D}}(K_x) = \int_{-\infty}^{\infty} P_{\mathrm{2D}}(K_x, K_y)\, dK_y \,. \qquad (7)$$

Surface realizations obtained using $P_{\mathrm{1D}}(K_x)$ are equivalent to 1-D cuts made through 2-D surface realizations obtained using $P_{\mathrm{2D}}(\boldsymbol{K})$.

For the case of the Pierson-Moskowitz spectrum the integral in (7) does not reduce to a convenient analytic form. While (7) could be evaluated numerically, we will instead use an approximation to the Pierson-Moskowitz spectrum that will yield an analytic form for the 1-D spectrum.

We approximate (5) by

$$\begin{aligned} P_{\mathrm{2D}}(\boldsymbol{K}) &= \frac{\alpha}{4\pi k^4}, \qquad K > K_L \\ P_{\mathrm{2D}}(\boldsymbol{K}) &= 0, \qquad K < K_L\,. \end{aligned} \qquad (8)$$

Using (4) one finds the same rms height as for the original Pierson-Moskowitz spectrum. Then using (8) in (7) yields the following 1-D roughness spectrum:

$$\begin{aligned} P_{\mathrm{1D}}(K_x) &= \frac{\alpha}{8\,|K_x|^3} \quad \text{for } |K_x| > K_L, \\ P_{\mathrm{1D}}(K_x) &= \frac{\alpha}{8\,|K_x|^3} F(K_x) \quad \text{for } |K_x| < K_L, \end{aligned} \qquad (9)$$

where

$$F(K_x) = \frac{2}{\pi}[\arcsin(|K_x|/K_L) - |K_x|\sqrt{K_L^2 - K_x^2}/K_L^2]. \qquad (10)$$

For $K_x \to 0$ it is necessary to expand (10) yielding

$$F(K_x) = \frac{2}{\pi}\frac{|K_x|^3}{K_L^3}\left(\frac{2}{3}+\frac{1}{5}\frac{K_x^2}{K_L^2}\right) \qquad (11)$$

for use in (9).

The 1-D roughness spectrum given by (9)-(11) has been used recently to study the accuracy of the SSA result for the coherent surface reflection coefficient [JASA **116**, 1975-1984 (2004)].

For a 1-D rough surface (the 2-D reverberation problem), the surface scattering strength is given by

$$SS = 10\log_{10}\sigma_{1\mathrm{D}}\ , \qquad (12)$$

where $\sigma_{1\mathrm{D}}$ is the bistatic scattering cross section per unit surface length per unit scattering angle and is dimensionless.

The perturbation theory expression for the bistatic and monostatic scattering cross section is given by

$$\sigma_{1\mathrm{D}} = \frac{4k_{iz}^2 k_{sz}^2}{k} P_{1\mathrm{D}}(k_{ix}-k_{sx}) = 4k^3\sin^2\theta_i\sin^2\theta_s P_{1\mathrm{D}}(k_{ix}-k_{sx}), \qquad (13)$$

where for bistatic backscattering

$$k_{ix}-k_{sx} = k(\cos\theta_i + \cos\theta_s). \qquad (14)$$

Surface coherent reflection loss for 1-D surfaces

The surface coherent reflection loss in dB is given by

$$RL = -20\log_{10}(|R_A|), \qquad (15)$$

where R_A is the amplitude coherent surface reflection coefficient, and in general is complex. The small slope approximation yields a series of expressions for R_A, and greater accuracy is obtained by going beyond the lowest-order expression. (See JASA **116**, 1975-1984 (2004) for more discussion and additional references.) We will denote the lowest-order approximation scheme by SSA(1) and the next order by SSA(2). When using SSA(1), scattering results reduce correctly to lowest-order perturbation theory in the small roughness regime, and when using SSA(2), the reduction is correct to second-order in perturbation theory. In the perturbation theory result for the coherent reflection coefficient, the first correction to the flat surface reflection coefficient comes in at second-order in surface height, which is an important reason to consider going beyond

SSA(1) to obtain R_A. In contrast to perturbation theory, SSA expressions for R_A are not restricted to the small roughness regime.

The SSA(1) result for R_A is the same as the familiar Kirchhoff approximation expression and is given by

$$\text{SSA(1)}: \; R_A = -\exp(-2k^2h^2\sin^2\theta_i). \qquad (16)$$

The SSA(2) result for R_A is given by

$$\begin{aligned}\text{SSA(2)}: \; R_A = -\exp(-2k^2h^2\sin^2\theta_i)[-1-2k^2h^2\sin^2\theta_i \\ +2k\sin\theta_i\int_{-\infty}^{\infty} dk_1 P_{1\mathrm{D}}(k\cos\theta_i - k_1)\sqrt{k^2-k_1^2}\,].\end{aligned} \qquad (17)$$

Comparisons with rough surface PE simulations for 1-way propagation at 3.2 kHz and with a wind speed of 10 m/s show (17) to be superior to (16) [JASA **116**, 1975-1984 (2004)]. Using (16) in GRAB led to a propagated intensity that is high by just less then 1 dB at a range of 20 km, while with (17) the error was found to be negligible.

Files of R_A versus grazing angle from 0° to 90° for a wind speed of 10 m/s and for frequencies of 250, 1000, and 3500 Hz will be provided by Kevin Williams at APL-UW.

Surface coherent reflection loss for 2-D surfaces

For 2-D surfaces, the surface coherent reflection loss in terms of R_A is again given by (15). The SSA(1) result for R_A is also still given by (16). The SSA(2) result for R_A is given by

$$\begin{aligned}\text{SSA(2)}: \; R_A = -\exp(-2k^2h^2\sin^2\theta_i)[-1-2k^2h^2\sin^2\theta_i \\ +2k\sin\theta_i\int d^2K_1 P_{2\mathrm{D}}(\boldsymbol{K_i}-\boldsymbol{K_1})\sqrt{k^2-|\boldsymbol{K_1}|^2}\,].\end{aligned} \qquad (18)$$

Results using (18) are not presently available, and at this time we recommend use of (16) for 2-D surfaces. Results using (18) may become available later this year.

Bottom bistatic scattering for 2-D surfaces

For scattering from 2-D bottom roughness, we write the bistatic scattering strength as

$$SS = 10\log_{10}\sigma_{2\mathrm{D}}\,, \qquad (19)$$

where $\sigma_{2\mathrm{D}}$ is the bistatic scattering cross section per unit area per unit solid angle and is dimensionless.

The (lowest-order) perturbation theory expression for the bistatic scattering cross section is given by [see JASA **96**, 1748-1754 (1994); JASA **103**, 275-287 (1998)]

$$\sigma_{2\mathrm{D}} = \frac{k_1^4}{4}\{|a(\boldsymbol{K}_s,\boldsymbol{K}_i)[1+\Gamma(\boldsymbol{K}_s)][1+\Gamma(\boldsymbol{K}_i)] + b(\boldsymbol{K}_s,\boldsymbol{K}_i)[1-\Gamma(\boldsymbol{K}_s)][1-\Gamma(\boldsymbol{K}_i)]|^2\}P_{2\mathrm{D}}(\boldsymbol{K}_i-\boldsymbol{K}_s). \quad (20)$$

In this expression, $k_1 = 2\pi/\lambda_1$ is the (real) acoustic wavenumber in the water, the wavelength in the water is $\lambda_1 = c_1/f$ where c_1 is the water sound speed and f is the frequency.

As before we write the acoustic wave vector as

$$\boldsymbol{k} = \boldsymbol{K} + k_z\hat{\boldsymbol{z}}\ , \qquad (21)$$

where $\boldsymbol{K}$ denotes the horizontal component of the wave vector. Also in (20) $\Gamma(\boldsymbol{K})$ is the flat bottom amplitude reflection coefficient for a plane wave with horizontal wave vector component $\boldsymbol{K}$ and is given by

$$\Gamma(\boldsymbol{K}) = \frac{\rho\beta_1(\boldsymbol{K}) - \kappa\beta_2(\boldsymbol{K})}{\rho\beta_1(\boldsymbol{K}) + \kappa\beta_2(\boldsymbol{K})}\ , \qquad (22)$$

where $\rho = \rho_2/\rho_1$ and $\kappa = k_2/k_1$ with ρ_2 and ρ_1 the sediment and water density, respectively, and k_2 the (complex) wavenumber in the sediment. In (22)

$$\beta_1(\boldsymbol{K}) = \sqrt{1-|\boldsymbol{K}|^2/k_1^2} \qquad (23)$$

and

$$\beta_2(\boldsymbol{K}) = \sqrt{1-|\boldsymbol{K}|^2/k_2^2}\ . \qquad (24)$$

The square root in (23) is chosen so that β_1 is either positive or positive imaginary. The square root in (24) is chosen so that β_2 is in the first quadrant of the complex plane. For a plane wave in the water propagating at grazing angle θ, the z component of the wave vector is given by

$$k_{1z} = k_1\beta_1(\boldsymbol{K}) = k_1\sin\theta \qquad (25)$$

and the magnitude of the horizontal component is

$$|\boldsymbol{K}| = K = k_1\cos\theta \qquad (26).$$

Thus, the magnitudes of the horizontal wave vectors appearing in (20) readily follow from the incident and scattered grazing angles θ_i, θ_s.

We write the complex wavenumber in the sediment as

$$k_2 = k_{2r} + ik_{2i} = k_{2r}(1 + i\delta) \quad (27)$$

so that $\delta = k_{2i}/k_{2r}$ and $k_{2r} = 2\pi/\lambda_2$ with $\lambda_2 = c_2/f$. Here, λ_2 is the wavelength in the sediment, c_2 is the sound speed in the sediment, and f is the frequency. The attenuation in dB for propagation in the sediment to range r is

$$\text{atten(dB)} = -20\log_{10}[\exp(-k_{2i}r)] = 20\,k_{2i}\,r\log_{10}e = 20\,\delta k_{2r}\,r\log_{10}e. \quad (28)$$

The attenuation per wavelength in the sediment is then

$$\text{atten(dB}/\lambda_2) = 40\pi\,\delta\log_{10}e. \quad (29)$$

An attenuation of 0.5 dB/λ_2 then implies

$$\delta = \frac{1}{80\pi\log_{10}e} = 0.009162. \quad (30)$$

Next, in (20)

$$a(\boldsymbol{K_s},\boldsymbol{K_i}) = (\frac{1}{\rho} - 1)\frac{\boldsymbol{K_s}\cdot\boldsymbol{K_i}}{k_1^2} + 1 - \frac{\kappa^2}{\rho} = (\frac{1}{\rho} - 1)\cos\theta_i\cos\theta_s\cos\phi + 1 - \frac{\kappa^2}{\rho} \quad (31)$$

with ϕ the azimuthal bistatic angle and

$$b(\boldsymbol{K_s},\boldsymbol{K_i}) = \beta_1(\boldsymbol{K_s})\beta_1(\boldsymbol{K_i})(\rho - 1) = \sin\theta_s\sin\theta_i(\rho - 1). \quad (32)$$

Finally in (20), $P_{2\text{D}}(\boldsymbol{K})$ is the 2-D bottom roughness spectrum, normalized such that the mean square roughness height h^2 is given by

$$h^2 = \int P_{2\text{D}}(\boldsymbol{K})d^2K. \quad (33)$$

The form to be used for $P_{2\text{D}}(\boldsymbol{K})$ is given by

$$P_{2\text{D}}(\boldsymbol{K}) = \frac{h^2l^2}{2\pi(1 + K^2l^2)^{3/2}} = \frac{h^2K_L}{2\pi(K_L^2 + K^2)^{3/2}} \quad (34)$$

where $K_L \equiv 1/l$. It is easy to show that (33) and (34) are consistent. Two sets of bottom roughness parameters will be used: one set for a “rough bottom” and one set for a "typical sand bottom." For the rough bottom case, we take $K_L = 0.1\text{m}$, implying $l = 10\,\text{m}$, and we take $h = \sqrt{2}\,(0.1\text{m}) = 0.141\text{m}$. In the power law region where $K >> K_L$ (34) reduces to

$$P_{\text{2D}}(\boldsymbol{K}) = \frac{h^2 K_L}{2\pi K^3} \qquad (35)$$

and for this case the "spectral strength" is $h^2 K_L/(2\pi) = 3.18\times 10^{-4}\,\text{m}$. While the rms height would appear modest, the spectral strength here is relatively high at wavelength scales in comparison to typical sandy bottoms. This will lead to substantial scattering from the bottom.

The second set of bottom roughness parameters have been chosen to represent a "typical sand bottom" condition where the spectral level is reduced by a factor of 8 and the parameter l in (34) is increased from 10 m to 400 m. This will produce bottom roughness conditions much closer to those typically found for sandy bottoms. These changes lead to an increase in the mean square height by a factor of 5 or an increase in the rms height h by the square root of 5 so that h becomes 0.316 m. In this second roughness condition the rms height has increased, but this increase occurs at large spatial scales, leading to a smoother interface with large-scale undulations.

To summarize, we now have two bottom conditions:

Rough bottom: $P_{\text{2D}}(\boldsymbol{K})$ given by (34) with h = 0.141 m and K_L = 0.1 m^{-1}.

Typical sand bottom: $P_{\text{2D}}(\boldsymbol{K})$ given by (34) with h = 0.316 m and K_L = 2.5×10^{-3} m^{-1}.

These two bottom conditions will lead to quite different propagation conditions in a waveguide.

When the argument of $P_{\text{2D}}(\boldsymbol{K})$ is $\boldsymbol{K_i} - \boldsymbol{K_s}$ as in (20), then K^2 in (34) is replaced by

$$k^2[(\cos\theta_i - \cos\theta_s \cos\phi)^2 + \cos^2\theta_s \sin^2\phi]$$

where ϕ is the asimuthal angle between $\boldsymbol{K_i}$ and $\boldsymbol{K_s}$. Thus, for specular scattering, $\phi = 0$, and for in-plane bistatic backscattering, $\phi = \pi$. Therefore, for in-plane bistatic backscattering, K^2 in (34) is replaced by $k^2(\cos\theta_i + \cos\theta_s)^2$ and for monostatic backscattering by $4k^2\cos^2\theta_i$.

Bottom bistatic scattering for 1-D surfaces

The first step is to obtain a 1-D roughness spectrum. In this case the prescription given by (7) is readily carried out yielding

$$P_{1\mathrm{D}}(K_x)=\frac{h^2K_L}{\pi(K_L^2+K_x^2)}. \qquad (36)$$

Again, we have two bottom conditions:

Rough bottom: $P_{1\mathrm{D}}(K_x)$ given by (36) with $h = 0.141$ m and $K_L = 0.1$ m^{-1}.

Typical sand bottom: $P_{1\mathrm{D}}(K_x)$ given by (36) with $h = 0.316$ m and $K_L = 2.5\times10^{-3}$ m^{-1}.

For a 1-D rough surface (the 2-D reverberation problem), the bottom scattering strength is given by

$$SS=10\log_{10}\sigma_{1\mathrm{D}}\ , \qquad (37)$$

where $\sigma_{1\mathrm{D}}$ is the bistatic scattering cross section per unit surface length per unit scattering angle and is dimensionless. The perturbation theory expression for the bistatic scattering cross section is given by

$$\begin{aligned}\sigma_{1\mathrm{D}}=\frac{k_1^3}{4}\{|\,&a(k_{sx},k_{ix})[1+\Gamma(k_{sx})][1+\Gamma(k_{ix})]\\&+b(k_{sx},k_{ix})[1-\Gamma(k_{sx})][1-\Gamma(k_{ix})]\,|^2\}P_{1\mathrm{D}}(k_{ix}-k_{sx}),\end{aligned} \qquad (38)$$

where

$$a(k_{sx},k_{ix})=-(\frac{1}{\rho}-1)\cos\theta_i\cos\theta_s+1-\frac{\kappa^2}{\rho} \qquad (39)$$

and

$$b(k_{sx},k_{ix})=\sin\theta_s\sin\theta_i(\rho-1). \qquad (40)$$

The reflection coefficients in (38) are given by (22). For bistatic backscattering

$$k_{ix}-k_{sx}=k(\cos\theta_i+\cos\theta_s). \qquad (39)$$

Bottom coherent reflection loss for 1-D surfaces

The bottom coherent reflection loss in dB is given by

$$RL=-20\log_{10}(|R_A|), \qquad (42)$$

where R_A is the amplitude coherent bottom reflection coefficient, and is complex. As with the surface case, the small slope approximation yields a series of expressions for R_A, and greater accuracy is obtained by going beyond the lowest-order expression. We will denote the lowest-order approximation scheme by SSA(1) and the next order by SSA(2). When using SSA(1), scattering results reduce correctly to lowest-order perturbation theory in the small roughness regime, and when using SSA(2), the reduction is correct to second-order in perturbation theory.

The SSA(1) result for R_A is the same as the Kirchhoff approximation expression and is given by

$$\text{SSA(1)}: \; R_A = \Gamma(k_{ix}) \exp(-2k^2 h^2 \sin^2 \theta_i) . \qquad (43)$$

The SSA(2) result for R_A is similar in form to the surface result given by (17), except the factor multiplying the roughness spectral density function inside the integral is much more complicated and will not be given here.

Comparisons with rough bottom PE simulations for 1-way propagation at 3 kHz show that the SSA(1) result given by (43) is perfectly adequate for the bottom coherent reflection loss for the "typical sand bottom" case. However, when the bottom roughness is increased to be comparable to the "rough bottom" case, the SSA(2) result would be preferred.

Files of SSA(2) results for R_A versus grazing angle from 0° to 90° for the rough bottom case for frequencies of 250, 1000, and 3500 Hz will be provided by Kevin Williams at APL-UW.

Note: While the bottom coherent reflection coefficient is correctly given by (43) for the "typical sand bottom" case, one-way propagation simulations by APL-UW indicate that the use of this reflection coefficient may be inappropriate for reverberation modeling for the typical sand bottom case. For this case, the scattered incoherent energy remains closely confined about reflected rays. Thus, the incoherent energy is not removed from the waveguide significantly more than reflected rays and can therefore continue to contribute to reverberation. In other words, the flat bottom reflection coefficient may be more appropriate for reverberation modeling than (43) for the typical sand bottom case. This remark does not apply to the rough bottom case.

Bottom coherent reflection loss for 2-D surfaces

For 2-D surfaces, the bottom coherent reflection loss in terms of R_A is again given by (42). The SSA(1) result for R_A is also still given by (43).

Results using the SSA(2) result for R_A are not presently available, and at this time we recommend use of (43) to obtain the bottom coherent reflection loss for 2-D surfaces for both models of bottom roughness. Results using SSA(2) may become available later this year.

Note: While the bottom coherent reflection coefficient is correctly given by (43) for the "typical sand bottom" case, one-way propagation simulations by APL-UW indicate that the use of this reflection coefficient may be inappropriate for reverberation modeling for the typical sand bottom case. For this case, the scattered incoherent energy remains closely confined about reflected rays. Thus, the incoherent energy is not removed from the waveguide significantly more than reflected rays and can therefore continue to contribute to reverberation. In other words, the flat bottom reflection coefficient may be more appropriate for reverberation modeling than (43) for the typical sand bottom case. This remark does not apply to the rough bottom case.